data are presented in Table IV. In this case, the difference between Z_{step} and Z_{mean} increases, especially for the thicker conductors.

3) The inner conductor is situated asymmetrically in the square shield—$B=1$, $S=D=0.2$. As follows from Table V, the effectiveness of the proposed method is greater for the thicker conductors.

IV. Conclusion

The numerical data presented in Tables I–V show that the utilization of the SCFM with the step current density approximation makes it possible to calculate the characteristic impedance of a different TEM transmission line with rectangular shape of the conductors with good accuracy. For the lines with symmetry, even the one step approximation is enough, while the general case of an asymmetrical position of the thick inner conductor into the shield needs the utilization of the step current densities.

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Definition of Nonlinear Reflection Coefficient of a Microwave Device Using Describing Function Formalism

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***Abstract*—At microwaves, it is necessary to define rigorously the large signal reflection coefficient of a nonlinear device. In this paper, the describing function concept is applied to the power waves incident on, and reflected by, a nonlinear element.**

This method allows us to define the nonlinear reflection coefficient (NLRC) on the power wave basis.

This NRLC is then compared with that defined on the current or voltage basis.

Numerical calculations applied to nonlinear elements illustrate the theoretical results.

I. Introduction

To use nonlinear devices, one might generalize linear concepts such as impedance, admittance, and transfer function by the so-called describing function method [1], [2]. These quantities would be defined for given input signals.

At microwaves, generally, the quantity measured is the reflection coefficient (or S-parameters for n-port devices). So it is necessary to define exactly the nonlinear reflection coefficient concept and its relation with the nonlinear impedance.

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In this short paper, we define the nonlinear reflection coefficient by application of the describing function (DF) method to the power waves, then we compare the nonlinear reflection coefficient (NLRC) and the nonlinear impedance of the same device. Some numerical results concerning nonlinear elements will be given.

At microwave frequencies, what is measured is the power of reflected or incident waves; thus, one obtains by direct measurement the reflection coefficient (S-parameters) [5]. So we should define the describing function in terms of incident and reflected waves. The definitions of the incident and reflected waves at the device terminals are

$$a(t)=\frac{v(t)+Z_0 i(t)}{2\sqrt{Z_0}} \tag{1}$$

$$b(t)=\frac{v(t)-Z_0 i(t)}{2\sqrt{Z_0}} \tag{2}$$

where Z_0 is the reference impedance, and i and v are instantaneous current and voltage in the device.

In a linear circuit, the reflection coefficient is defined as

$$\Gamma(\omega)=\frac{\mathrm{F}\{b(t)\}}{\mathrm{F}\{a(t)\}} \tag{3}$$

where F stands for Fourier Transform which can be also expressed as

$$\Gamma(\omega)=\frac{Z(\omega)-Z_0}{Z(\omega)+Z_0}=\frac{Y_0-Y(\omega)}{Y_0+Y(\omega)} \tag{4}$$

but this does not hold for the nonlinear case.

II. Nonlinear Resistance

Let us suppose that the instantaneous relation between i and v (for a purely resistive device) is $i=f(v)$.

Substituting into [1] and [2], we obtain

$$a(t)=\frac{v+Z_0 f(v)}{2\sqrt{Z_0}}$$

$$b(t)=\frac{v-Z_0 f(v)}{2\sqrt{Z_0}}. \tag{5}$$

b and a are parametrically related; one can then draw the characteristic curve and obtain

$$b=g(a). \tag{6}$$

By application of the describing function method, one can seek a linear approximation of this relation by putting

$$\int_0^T \frac{\partial}{\partial \Gamma_{\mathrm{NL}}}\{g(a)-\Gamma_{\mathrm{NL}}\cdot a)\}^2\,dt=0. \tag{7}$$

Γ_{NL} is immediately deduced as

$$\Gamma_{\mathrm{NL}}=\frac{\int_0^T a g(a)\,dt}{\int_0^T a^2\,dt} \tag{8}$$

and one can write

$$B=\Gamma_{\mathrm{NL}} A$$

where B and A are amplitudes of the reflected and incident waves.

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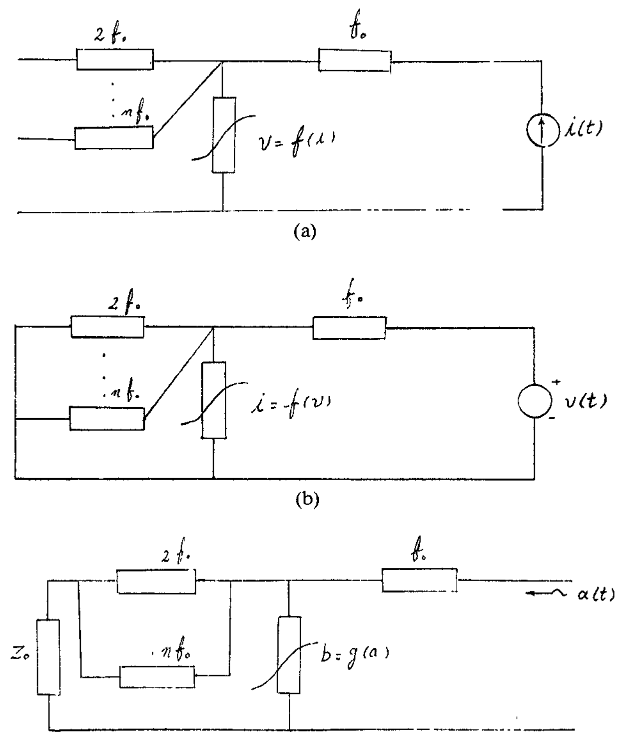

Fig. 1. (a) Schematic representation of the nonlinear resistance describing function mechanism. (b) Schematic representation of the nonlinear conductance describing function mechanism. (c) Schematic representation of the nonlinear reflection coefficients describing function mechanism.

It should be noted that this definition for the nonlinear reflection coefficient supposes that all harmonics of the reflected wave are loaded by the characteristic impedance Z_0 (see Fig. 1 in which the blocks are bandpass filters centered at the frequencies indicated).

Equations (5) through (8) represent the basis for the calculation of the NLRC. Moreover, (5) shows that if $v(t)$ is sinusoïdal, $a(t)$ will not be sinusoïdal. We can then define another nonlinear reflection coefficient on a sinusoïdal input voltage basis; that is, first calculate the effective conductance Y_e (or resistance), and then put

$$\Gamma'_{NL} = \frac{Y_0 - Y_e}{Y_0 + Y_e}. \tag{9}$$

Obviously, this reflection coefficient would be different from NLRC since in the first one we suppose $v(t)$ sinusoïdal and in the latter case we suppose $a(t)$ sinusoïdal. A practical case in which Γ_{NL} is useful is that of a microwave oscillator in which the nonlinear device is loaded, through a transmission line coupled to a dielectric resonator, by the characteristic impedance Z_0. Here, obviously, all harmonics of the reflected wave are loaded by Z_0.

III. Numerical Results

Several nonlinear resistances and capacitances have been tried to find the NLRC, of which two examples are given.

A. Nonlinear Reflection Coefficient of an NL Conductance

Let us take for example a tunnel diode whose characteristic is given by

$$i(v) = C_1\left\{(V_T - v)^2 u(V_T - v) + C_2\right\} \tanh C_3 v + C_4 \exp C_5 v \tag{10}$$

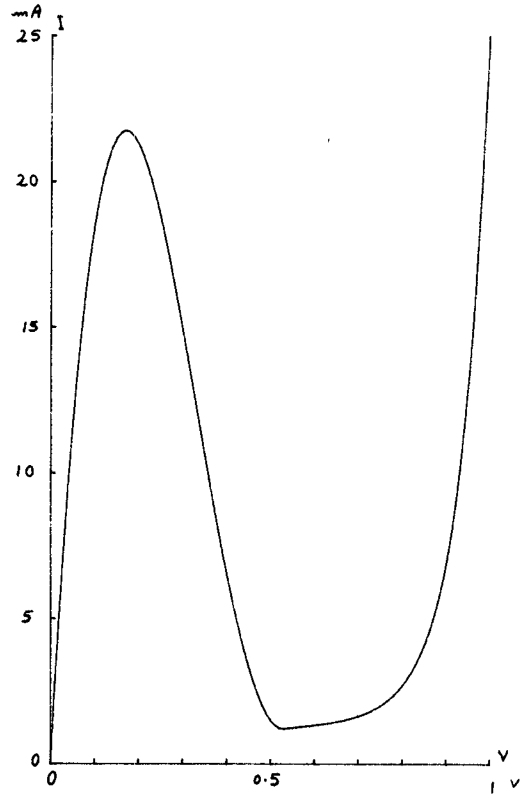

Fig. 2. The tunnel diode $I - V$ characterisitc.

where $u(x)$ is the unit step function and coefficients V_T and C_1 to C_5 are taken so as to fit a given $I - V$ characteristic (see Fig. 2). In our example, the following numerical values have been taken:

$$V_T = 0.53 \quad C_1 = 0.5 \quad C_2 = 0.003$$
$$C_3 = 2 \quad C_4 = 1 \times 10^{-8} \quad C_5 = 14.7.$$

The diode is biased at $V_0 = 0.3$ V.

Then the relation $b = g(a)$ is calculated numerically, and for a sinusoïdal incident wave, Γ_{NL} is calculated from (8) (see Fig. 3).

The effective conductance is calculated from the relation

$$G_e = \frac{\int_0^T iv\,dt}{\int_0^T v^2\,dt} \tag{11}$$

and is given as a function of V_0, the amplitude of sinusoïdal input voltage. Then the NL reflection coefficient, on the sinusoïdal input voltage basis, would be

$$\Gamma'_{NL} = \frac{Y_0 - G_e}{Y_0 + G_e}.$$

$\Gamma_{NL} - A$ and $\Gamma'_{NL} - A$ curves are then compared (A is the amplitude of the incident wave).

B. Nonlinear Reflection Coefficient of a Nonlinear Capacitance

The equation characterizing a capacitance is a $q - v$ relation (see Fig. 4), i.e.,

$$q = f(v). \tag{12}$$

Suppose that this is given by a polynomial approximate form

$$q = \alpha_1 v + \alpha_2 v^2 + \alpha_3 v^3 + \cdots. \tag{13}$$

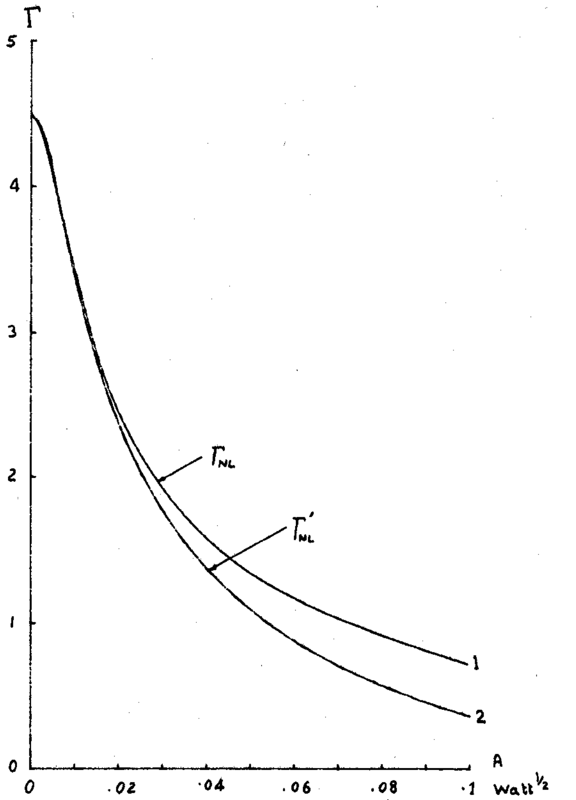

Fig. 3. Comparison between the two nonlinear reflection coefficients calculated for the tunnel diode. 1) Obtained from $b-a$ linearization. 2) Obtained from the effective conductance.

Now, the describing function of q is immediately found to be

$$q_{NL} = \left(\alpha_1 + \alpha_2 \frac{\overline{V^3}}{\overline{V^2}} + \alpha_3 \frac{\overline{V^4}}{\overline{V^2}} + \cdots \right) v \tag{14}$$

where

$$\overline{V^n} = \frac{1}{T}\int_0^T v^n \, dt.$$

For a sinusoïdal input voltage $v = V_0 \sin \omega t$

$$q_{NL} = \left(\alpha_1 + \frac{3}{4}\alpha_3 V_0^2 + \cdots \right) v \tag{15}$$

and

$$i = \frac{dq_{NL}}{dt} = \left(\alpha_1 + \frac{3}{4}\alpha_3 V_0^2 + \cdots \right) \frac{dv}{dt}$$

$$i = \left(\alpha_1 + \frac{3}{4}\alpha_3 V_0^2 + \cdots \right) V_0 \omega \cos \omega t. \tag{16}$$

To find the reflection coefficient we calculate a and b

$$a(t) = \frac{V_0}{2\sqrt{Z_0}\cos\phi} \sin(\omega t + \phi) \tag{17}$$

by the same way

$$b(t) = \frac{V_0}{2\sqrt{Z_0}\cos\phi} \sin(\omega t - \phi) \tag{18}$$

where

$$\phi = \tan^{-1} Z_0 \omega \left(\alpha_1 + \frac{3}{4}\alpha_3 V_0^2 + \cdots \right). \tag{19}$$

Fig. 4. Phase shift of the reflected wave from the nonlinear capacitance as calculated by two methods. 1) $Q-V$ linearization. 2) Direct solution of the differential equation.

Then

$$\Gamma'_{NL} = \frac{B}{A} = 1 \angle -2\phi.$$

This is the nonlinear reflection coefficient for a sinusoïdal input (*its amplitude is always equal to unity*).

Sinusoïdal incident wave; $a(t) = A \cdot \sin \omega t$: The current in the nonlinear capacitance is

$$i = f'(v)\frac{dV}{dt} \tag{20}$$

given

$$i = \frac{a-b}{\sqrt{Z_0}}$$

and

$$v = (a+b)\sqrt{Z_0}.$$

Inserting into (20), we obtain

$$\frac{db}{dt} = \frac{a-b}{Z_0 f'\{(a+b)\sqrt{Z_0}\}} - \frac{da}{dt}. \tag{21}$$

This is a simple differential equation of the form $y' = f(x,y)$, which can be solved numerically to find $b(t)$. The phase between the incident and the periodic reflected wave gives the phase angle of the reflection coefficient Γ_{NL}. Its modulus is given by

$$|\Gamma_{NL}| = \frac{\text{Amplitude of } b(t) \text{ fundamental}}{\text{Amplitude of } a(t)}$$

$$|\Gamma_{NL}| = \frac{2}{TA_0}\int_{t_1}^{T+t_1} b(t) \sin \omega t \, dt. \tag{22}$$

C. Numerical Example

A nonlinear capacitance of the Schottky barrier is taken with the

$$Q=\sqrt{2\epsilon q N_d (V_0 - V)} \tag{23}$$

numerical example: $Q = 2.06\times10^{-12}\sqrt{0.5-V}$ biased at $V=-10$ V at 10 GHz.

IV. Conclusion

A definition of the nonlinear reflection coefficient based on the application of the describing function to the power waves has been proposed in this short paper.

Care must be taken to determine in a circuit configuration which is the input waveform: sinusoïdal input current, sinusoïdal input voltage, or sinusoïdal incident wave. As shown in the example, discrepancies might arise between different cases. As we have investigated by computer simulation for nonlinear elements with odd symmetry about the operating point, these discrepancies are small, while for nonlinear elements without symmetry (tunnel diode in the example) they are quite important. This concludes that while working on a network analyzer one might interchange nonlinear resistance and nonlinear reflection coefficient concepts in the first case, while in the latter case nonlinear reflection coefficient on a $b-a$ linearization basis should be used, provided "b" harmonics are loaded by the characteristic impedance.

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Application of Boundary-Element Method to Electromagnetic Field Problems

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Abstract—**This paper proposes an application of the boundary-element method to two-dimensional electromagnetic field problems. By this method, calculations can be performed using far fewer nodes than by the finite-element method, and unbounded field problems are easily treated without special additional consideration. In addition, the results obtained have fairly good accuracy. In this paper, analyzing procedures of electromagnetic field problems by the boundary-element method, under special conditions, are proposed and several examples are investigated.**

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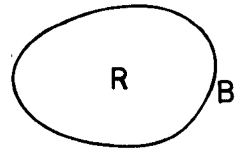

Fig. 1. Two-dimensional region R.

I. Introduction

At present, the finite-element method is widely used in many fields. The main reason may be that, by the finite-element method, it is easy to handle inhomogeneities and complicated structures. However, it requires a large computer memory and long computing time to solve the final matrix equation. In addition, unbounded field problems need some additional techniques [1], [2].

Recently, the boundary-element method has been proposed, which is interpreted as a combination technique of the conventional boundary-integral equation method and a discretization technique, such as the finite-element method, and which has merits of both the above methods, i.e., the required size of the computer memory being small and the obtained results having fairly good accuracy [3], [4]. Namely, the boundary-element method is a boundary method and, therefore, if the region to be analyzed is homogeneous, then it requires nodes, necessary for calculation, on its boundary only. So the problem can be treated with one less dimension. Moreover, it can handle unbounded field problems easily, so that it is suitable for the electromagnetic field analysis which often includes unbounded regions [5], [6].

In this paper, a formulation of two-dimensional electromagnetic field problems by the boundary-element method and its application to several interesting cases, such as the problem of electromagnetic waveguide discontinuities, multi-media problems, and electromagnetic wave scattering problems [6]. In addition, several examples are analyzed and the results obtained with the boundary-element method are compared with rigorous ones, and solutions of the other numerical methods. The propriety of our analyzing procedure of the boundary-element method is verified.

II. General Formulation

A two-dimensional region R enclosed by a boundary B, as illustrated in Fig. 1, is considered. In the region R, Helmholtz's equation

$$(\nabla^2 + k^2)u = 0 \tag{1}$$

holds, where u is the potential used for analysis, we write its outward normal derivative as q, and k denotes the wavenumber in free space. The boundary condition on B is

$$u = \bar{u} \tag{2}$$

or

$$q = \bar{q} \tag{3}$$

where "$\bar{\ }$" means a known value. Here, Green's function

$$u^* = -\frac{j}{4}H_0^{(2)}(kr) \tag{4}$$

is introduced, where $H_0^{(2)}$ is the Hankel function of the second kind and order zero. By the method of weighted residuals [3], [4] or Green's formula, the following equation is obtained:

$$u_i + \int_B u q^*\, dc = \int_B q u^*\, dc. \tag{5}$$